
Improved Algorithms for Fair Matroid Submodular Maximization

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Abstract

1 Submodular maximization subject to matroid constraints is a central problem with
2 many applications in machine learning. As algorithms are increasingly used in
3 decision-making over datapoints with sensitive attributes such as gender or race, it
4 is becoming crucial to enforce fairness to avoid bias and discrimination. Recent
5 work has addressed the challenge of developing efficient approximation algorithms
6 for fair matroid submodular maximization. However, the best algorithms known so
7 far are only guaranteed to satisfy a relaxed version of the fairness constraints that
8 loses a factor 2, i.e., the problem may ask for ℓ elements with a given attribute, but
9 the algorithm is only guaranteed to find $\lfloor \ell/2 \rfloor$. In particular, there is no provable
10 guarantee when $\ell = 1$, which corresponds to a key special case of perfect matching.
11 In this work, we achieve a new trade-off via an algorithm that gets arbitrarily close
12 to full fairness. Namely, for any constant $\varepsilon > 0$, we give a constant-factor approx-
13 imation to fair monotone matroid submodular maximization that in expectation
14 loses only a factor $(1 - \varepsilon)$ in the lower-bound fairness constraint. Our empirical
15 evaluation on a standard suite of real-world datasets – clustering, recommendation,
16 and coverage tasks – demonstrates the practical effectiveness of our methods.

17 1 Introduction

18 Machine learning is increasingly deployed in high-stakes decision-making, raising concerns about
19 the propagation of bias and unfairness in automated systems. These challenges are especially acute
20 in domains such as education, law enforcement, hiring, and credit [MMD16; Whi22; Eur22]. In
21 response, a growing body of research has focused on developing algorithms that incorporate fairness
22 constraints for core problems including clustering [CKLV17], data summarization [CKSDKV18],
23 classification [ZVGG17], voting [CHV18], and ranking [CSV18].

24 This paper studies fairness in the context of monotone submodular maximization subject to matroid
25 constraints. Submodular functions, which capture the principle of diminishing returns, are funda-
26 mental to a range of machine learning applications such as recommender systems [EG11], feature
27 selection [DK11], active learning [GK11], and data summarization [LB11]. Matroids provide a
28 general framework for modeling independence constraints, encompassing cardinality, partition, graph
29 connectivity, and linear independence constraints.

30 While numerous fairness definitions have been proposed, we adopt a widely used group fairness
31 model [CHV18; CKSDKV18; CSV18; CKLV17; CKLV19; EMNTT20; EFNTT23; ETNV24], which
32 partitions the universe into *disjoint* groups and enforces *lower and upper* bounds on the representation
33 of each sensitive group in the selected set. See Section 2.1 for a precise definition. This model
34 generalizes several fairness notions, such as proportional representation [Mon95; BLS17], diversity
35 constraints [CCRL13; Bid06], and statistical parity [DHPRZ12].

In the absence of fairness constraints, monotone submodular maximization under a single matroid constraint is very well understood, as a tight $(1 - 1/e) \approx 0.63$ -approximation is achievable [CCPV11; Fei98]. The intersection of two matroid constraints (which we refer to as “matroid intersection”) admits an almost 0.5-approximation [LSV10]. The fair variant has been primarily explored under cardinality constraints [CHV18], where a tight $(1 - 1/e)$ -approximation is also known. In the (single-pass) streaming setting, there is a 0.3178-approximation [FLNSZ22] for the non-fair matroid version; furthermore, since the intersection of cardinality constraint and fairness can be reduced to a single matroid constraint [EMNTT20], the same approximation factor can be obtained for it.

However, the intersection of a matroid constraint and a fairness constraint seems significantly more challenging, and is still poorly understood despite two recent works devoted to studying this problem in the streaming [EFNTT23] and the classic offline [ETNV24] settings; our focus is on the latter. Following [EFNTT23], we refer to the problem as Fair Matroid Monotone Submodular Maximization (**FMMSM**). To appreciate its difficulty, consider a key special case, Monotone Submodular Perfect Matching (**MSPM**), i.e., maximizing a monotone submodular function over the collection of all *perfect* matchings in a *bipartite* graph (V_G, E_G) .¹ This collection of feasible sets is not downward-closed, which invalidates known algorithmic approaches.² The best known approximation factor for MSPM is a trivial $O(|V_G|)$ -approximation; one can also apply the framework of [GHIM09] to obtain an $\tilde{O}(\sqrt{|E_G|})$ -approximation, which is superior for sparse graphs. In fact, this could possibly even be tight, as it almost matches a surprising negative result of [ETNV24] who showed a family of sparse graphs where the standard *multilinear relaxation* (commonly used in relax-and-round approaches for submodular optimization) has an integrality gap of $\Omega(\sqrt{|E_G|})$. The existence of a constant-factor approximation to MSPM was posed by [ETNV24] as an exciting open problem.

The algorithms given in [EFNTT23; ETNV24] for FMMSM circumvent the difficulty posed by the lower bound constraints by relaxing them. They obtain the following two results:

Theorem 1.1 (Two-pass algorithm of [EFNTT23]) *There is a polynomial-time algorithm for FMMSM that violates lower bound constraints by a factor 2 and obtains $\alpha/2$ -approximation, where α is the approximation ratio of an algorithm for maximizing a monotone submodular function under a matroid intersection constraint.*

We can have α be almost $1/2$ [LSV10] and thus get an almost $1/4$ -approximation. ([EFNTT23] work in the streaming setting and instead use the streaming algorithm for matroid intersection of [GJS21]; this results in a $1/11.66$ -approximation in two passes.) Here, violating lower bound constraints by a factor 2 means that, if a color has a lower bound of ℓ , the solution is guaranteed to have at least $\lfloor \ell/2 \rfloor$ elements of that color. Note that in MSPM we have $\ell = 1$ and thus $\lfloor \ell/2 \rfloor = 0$.

Theorem 1.2 ([ETNV24]) *There is a polynomial-time algorithm for FMMSM that satisfies lower and upper bound constraints in expectation rather than exactly, and obtains a $(1 - 1/e)$ -approximation in expectation.*

Theorem 1.2 also guarantees certain two-sided tail bounds on the violation of each fairness constraint which apply if ℓ is large enough. It is the only algorithm considered in this paper that violates the *upper* bounds. The algorithm proceeds by solving and rounding the multilinear relaxation.

If we consider a relaxed version of MSPM where instead of a *perfect* matching we want a *large* matching that also has high submodular function value, then a simple greedy algorithm will yield a $1/3$ -approximation (Theorem 2.5) and construct a maximal matching, thus getting $1/2$ of the maximum possible size. The results in Theorems 1.1 and 1.2 give no improvement upon this. While one can try to generalize this simple approach to FMMSM, it faces another issue that is salient in the context of fairness motivations: while at least half of the total lower bound mass will be satisfied, there could be “unlucky” colors (marginalized groups) that never get represented in the solution; this is precisely the reason why we seek fair algorithms in the first place.

¹To see why MSPM is a special case of FMMSM, set E_G as the universe, consider a partition matroid that encodes that every vertex on the left shall have degree at most 1 in the solution, and set fairness constraints so that every vertex on the right shall have degree at least 1 and at most 1.

²Of course, a proper subset of a perfect matching is not a perfect matching. But more importantly, the collection of all *subsets of perfect matchings* (which is downward-closed) does not belong to any of the families that are known to make approximate submodular maximization tractable. In particular, it is not a matroid, an intersection of few matroids, or a so-called p -extendible set system or a p -system [CCPV11] for $p = O(1)$.

83 1.1 Our contributions

84 In this work we provide an algorithm that satisfies the fairness constraints within a factor better than
 85 2, while also giving guarantees for every individual group rather than only in aggregate. To achieve
 86 the former, we trade off part of the objective value; to achieve the latter, we employ randomization.

87 **Theorem 1.3 (informal version of Theorem 3.4)** *For every $\varepsilon \in (0, 1)$ there is a polynomial-time*
 88 *algorithm for FMMSM whose output*

- 89 • *satisfies the matroid constraint,*
- 90 • *satisfies fairness upper bound constraints,*
- 91 • *for a group with fairness lower bound ℓ , has in expectation at least $(1 - \varepsilon)\ell$ elements from*
 92 *that group,*
- 93 • *has expected size at least $(1 - \varepsilon)$ times the maximum size of any feasible solution,*
- 94 • *satisfies Chernoff-style high-probability bounds on size, as well as total fairness violation,*
- 95 • *has expected submodular function value at least $0.499 \cdot \varepsilon \cdot \text{OPT}$.*

96 Our bound on the submodular function value is actually shown with respect to a more powerful
 97 optimum, namely, an optimal set that satisfies the matroid and upper-bound constraints, but not
 98 necessarily the lower-bound constraints. If one wants to compare to this optimum, then the $O(\varepsilon)$
 99 factor loss in value is unavoidable. To see this, consider MSPM in a graph $P_3 \times N$ consisting of a
 100 disjoint union of N paths of length 3, with a linear objective function assigning values 0, 1, 0 to each
 101 path’s edges. A perfect matching of size $2N$ has 0 value, and a maximal matching of size N has
 102 value N ; one can interpolate between these smoothly.

103 As a second contribution, we also employ our techniques to obtain a deterministic algorithm. There are
 104 several variants that we could formulate; we choose to show a general setting of matroid intersection,
 105 where the trade-off is between size and objective value. The relation to fairness is that an algorithm
 106 that finds a solution of maximum size that is an α -approximation to the objective value would imply
 107 an α -approximation algorithm for FMMSM (see [EFNTT23], Proposition C.6).

108 **Theorem 1.4** *For every $\varepsilon \in (0, 1)$ there is a deterministic polynomial-time algorithm for the problem*
 109 *of maximizing a monotone submodular function subject to two matroid constraints whose output*
 110 *has size at least $(1 - \varepsilon)$ times the maximum size of any feasible solution minus one, and obtains a*
 111 *$(0.499 \cdot \varepsilon)$ -approximation to the submodular function value.*

112 **Experimental results.** We show the effectiveness of our algorithm empirically against prior work
 113 and natural baselines on a suite of standard benchmarks. We measure the submodular objective
 114 value and total fairness violation. Our algorithms produce solutions whose value is competitive with
 115 the highest-value baseline, which completely ignores the lower bound constraints and accordingly
 116 has the highest fairness violations. In two out of three scenarios, our algorithms dominate prior
 117 work [EFNTT23]. Finally, a key strength of our approach is the flexibility given by ε , allowing users
 118 to tune the balance between utility and fairness.

119 **Our techniques.** Let us begin with the simple setting of perfect matchings (MSPM). Consider the
 120 symmetric difference of a high-value matching Y and a perfect matching P . This decomposes into a
 121 collection of vertex-disjoint alternating cycles and augmenting paths.

122 One possible algorithm is to ignore the cycles, and choose some of the augmenting paths to apply to
 123 Y , so that its size grows to at least $(1 - \varepsilon)|P|$. We can do this by computing the marginal contribution
 124 of the elements that Y would lose in each path, and taking the least damaging paths; by submodularity,
 125 this loses at most a $(1 - \varepsilon)$ fraction of value in Y .

126 While this does ensure a large matching, some ε fraction of vertices can still be “unlucky” and end
 127 up unmatched. Deterministically this would be hard to avoid (short of solving MSPM/FMMSM
 128 completely, with no fairness violation); our next idea is to choose the paths randomly in the above
 129 solution. This will work for MSPM, as long as we take care to select a $(1 - \varepsilon)$ fraction of the $|P| - |Y|$
 130 many augmenting paths, even if we already have $|Y| > (1 - \varepsilon)|P|$. Then every vertex that was not
 131 matched in Y has a $(1 - \varepsilon)$ probability of being matched in the new solution.

132 However, there are two main challenges when trying to generalize the above approach to matroid
 133 and fairness constraints. Firstly, having fairness bounds with $\ell_c < u_c$ means that Y can have fewer
 134 elements than P in some colors but more elements in other colors, and can even have $|Y| = |P|$
 135 while still violating many fairness lower bounds. This means that we need to find and apply not only
 136 augmenting paths, but also alternating paths that exchange an element of an oversaturated color for
 137 one of an undersaturated color, without increasing the solution size. We show that as long as the total
 138 fairness violation is large, there are many such disjoint paths, which implies that applying a random
 139 fraction of them still retains enough value.

140 The second, larger obstacle arises due to dealing with general matroids. We are able to use tools
 141 from matroid theory to show the existence of many disjoint alternating or augmenting paths in an
 142 appropriate matroid intersection exchange graph whose vertices correspond to elements of Y and P
 143 (which were edges in the case of MSPM). We need to carefully refine the paths via an asymmetric
 144 shortcutting process to ensure that applying them leaves the solution independent in the matroid while
 145 also not disrupting the counts of elements in the colors not being exchanged. Moreover, in general,
 146 multiple augmenting paths in matroids cannot be applied simultaneously. We deal with this using an
 147 iterative framework where we apply a single path, rebuild the exchange graph, and find a new large
 148 collection of disjoint paths; we then bound the loss in value after each step.

149 **Paper organization.** In Section 2 we introduce all necessary notation, definitions, and useful facts.
 150 In Section 3 we describe our algorithms and prove their properties. Section 4 is devoted to the
 151 experimental evaluation. We conclude and discuss the limitations and broader impact of our work in
 152 Section 5. Additional related work is discussed in Section 1.3 in the full version (in the supplementary
 153 materials), which also contains all omitted content and skipped proofs.

154 2 Preliminaries

155 We denote the symmetric difference $(X \setminus Y) \cup (Y \setminus X)$ of two sets X and Y by $X \Delta Y$.

156 **Submodular functions.** We consider functions $f : 2^V \rightarrow \mathbb{R}_+$ defined on a ground set V . We say that
 157 f is *submodular* if $f(Y \cup \{e\}) - f(Y) \geq f(X \cup \{e\}) - f(X)$ for any two sets $Y \subseteq X \subseteq V$ and
 158 any element $e \in V \setminus X$. Moreover, f is *monotone* if $f(Y) \leq f(X)$ for any two sets $Y \subseteq X \subseteq V$.
 159 We assume that f is given as an oracle that computes $f(S)$ for given $S \subseteq V$; we consider the running
 160 time of this oracle to be $O(1)$.

161 The following fact is folklore. We provide a proof in the full version (see the supplementary material).

162 **Fact 2.1** *Let f be a non-negative submodular function and $X_1, X_2, \dots, X_k \subseteq X$ be disjoint subsets*
 163 *of X . Then*

$$\sum_{i=1}^k f(X \setminus X_i) \geq (k-1)f(X).$$

165 **Matroids.** A *matroid* is a set family $\mathcal{I} \subseteq 2^V$ with the properties:

- 166 • *Downward-closedness:* if $X \subseteq Y$ and $Y \in \mathcal{I}$, then $X \in \mathcal{I}$;
- 167 • *Augmentation:* if $X, Y \in \mathcal{I}$ and $|X| < |Y|$, then there exists $e \in Y$ with $X + e \in \mathcal{I}$.

168 We abbreviate $X \cup \{e\}$ as $X + e$ and $X \setminus \{e\}$ as $X - e$. We assume that the matroid is given as an
 169 oracle that, for a given $S \subseteq V$, answers whether $S \in \mathcal{I}$; we consider the running time of this oracle
 170 to be $O(1)$. We say that a set $S \subseteq V$ is *independent* if $S \in \mathcal{I}$.

171 **Matroid exchange graph.** Let \mathcal{I} be a matroid on universe V and Y, Z be two independent sets.

172 **Definition 2.2** *We define the exchange graph for Y and Z with respect to \mathcal{I} as the bipartite graph*

$$(Y \setminus Z, Z \setminus Y, \{(y, z) : Y - y + z \in \mathcal{I}\}).$$

173 **Lemma 2.3 ([Sch03], Corollary 39.12a)** *If $|Y| = |Z|$, then the exchange graph for Y and Z with*
 174 *respect to \mathcal{I} contains a perfect matching.*

2.1 Fair Matroid Monotone Submodular Maximization (FMMSM)

The universe V is partitioned into C sets: $V = V_1 \cup V_2 \cup \dots \cup V_C$, where V_c denotes elements of color c . Every element has exactly one color. The set of colors is denoted by $[C] = \{1, 2, \dots, C\}$. For every color $c \in [C]$ we have *fairness bounds*: lower bound ℓ_c and upper bound u_c .

The set of upper bounds gives rise to a *partition matroid* that we will denote by \mathcal{U} . That is,

$$\mathcal{U} = \{S \subseteq V \mid |S \cap V_c| \leq u_c \ \forall c \in [C]\}.$$

It is well-known that such a collection of sets forms a matroid. We will call a set $S \in \mathcal{U}$ *upper-fair*.

If a set satisfies both the lower and the upper bounds, we say that it is *fair*. That is, we define the family of fair sets \mathcal{C} as follows:

$$\mathcal{C} = \{S \subseteq V \mid \ell_c \leq |S \cap V_c| \leq u_c \ \forall c \in [C]\}.$$

The FMMSM problem asks to find a set $S \in \mathcal{I} \cap \mathcal{C}$ (i.e., fair and independent S) that maximizes $f(S)$. We use OPT for the optimal value, i.e., $\text{OPT} = \max_{S \in \mathcal{I} \cap \mathcal{C}} f(S)$. We assume that there exists a fair and independent set, i.e., $\mathcal{I} \cap \mathcal{C} \neq \emptyset$. We say that an algorithm is an α -approximation if it outputs a set S with $f(S) \geq \alpha \cdot \text{OPT}$.

For any set $S \subseteq V$ we define its *fairness violation* $\text{fav}(S) := \sum_c \max\{|S \cap V_c| - u_c, \ell_c - |S \cap V_c|, 0\}$. Note that if S is upper-fair, then $\text{fav}(S) = \sum_c \max\{\ell_c - |S \cap V_c|, 0\}$.

Lemma 2.4 ([EFNTT23], Appendix C) *There is an exact polynomial-time algorithm for FMMSM for the case when f is a linear function.*

Matroid intersection. Given two matroids and a monotone submodular function f defined on V , we can define the problem of maximizing a submodular function subject to a matroid intersection constraint similarly to FMMSM. In particular, if we ignore the lower bounds completely, FMMSM turns into the above matroid intersection problem for matroids \mathcal{I} and \mathcal{U} .

Theorem 2.5 ([CCPV11]) *The greedy algorithm gives a $1/3$ -approximation to this problem.*

Theorem 2.6 ([LSV10]) *For any $\delta > 0$ there is a polynomial-time algorithm that gives a $(0.5 - \delta)$ -approximation to this problem.*

3 Our algorithm

In this section we describe our algorithms: randomized (Theorem 3.4) and deterministic (Section 3.1). We first need to introduce some notions. The proof of Theorem 3.4 will begin by constructing a maximum-cardinality independent and fair set P , which will stay unchanged throughout the execution. We also construct an independent and upper-fair set Y of high f -value. We will use P as a source of fairness and iteratively trade off Y 's value for P 's elements in colors that are undersaturated by Y .

Definition 3.1 *Given Y and P as above, we say that a color $c \in [C]$ is undersaturated if $|Y \cap V_c| < |P \cap V_c|$, and oversaturated if $|Y \cap V_c| > |P \cap V_c|$.*

The technical crux of the proof of Theorem 3.4 is Lemma 3.3, in which we show the existence of many disjoint structures, each of which can be used to advance our fairness objective.

Definition 3.2 *Let Y be an independent and upper-fair set, and let $X \subseteq V$. Define the result Y' of applying X to Y as the symmetric difference $Y' = Y \triangle X$. We say that X is alternating (with respect to Y) if Y' is independent ($Y' \in \mathcal{I}$) and there is exactly one undersaturated color $c' \in [C]$ and one oversaturated color $c'' \in [C]$ such that for all $c \in [C]$,*

$$|Y' \cap V_c| = |Y \cap V_c| + \begin{cases} 1 & \text{for } c = c', \\ -1 & \text{for } c = c'', \\ 0 & \text{for } c \neq c', c''. \end{cases}$$

We say that X is augmenting if all the above conditions are satisfied, except that there is no color c'' . In both cases, we say that X increases c' .

214 Note that we have $|Y'| = |Y|$ if X is alternating and $|Y'| = |Y| + 1$ if X is augmenting. Also, Y' is
 215 upper-fair, since the only color where it has more elements than Y is c' , and we have $|Y' \cap V_{c'}| =$
 216 $|Y \cap V_{c'}| + 1 < |P \cap V_{c'}| + 1$ (and P is fair).

217 **Lemma 3.3** *Let Y and P be two independent and upper-fair sets with $|Y| \leq |P|$. Denote*

$$k = \sum_{c \in [C]} \max(0, |P \cap V_c| - |Y \cap V_c|).$$

218 *Then we may find in polynomial time a collection X_1, \dots, X_k of disjoint subsets of $Y \cup P$, of which at*
 219 *least $|P| - |Y|$ many are augmenting and the rest are alternating. Moreover, for every undersaturated*
 220 *color c , exactly $|P \cap V_c| - |Y \cap V_c|$ many of the paths increase c .*

221 The full proof can be found in the full version (see the supplementary material).

222 *Proof sketch.* We consider the so-called (directed) matroid intersection exchange graph for Y and
 223 P with respect to \mathcal{I} and \mathcal{U} . Inside this graph we carefully construct a subgraph consisting of two
 224 matchings M_{\leftarrow} and M_{\rightarrow} . M_{\leftarrow} is obtained by invoking Lemma 2.3 on a subgraph, whereas we
 225 construct M_{\rightarrow} manually by matching elements of the same colors between Y and P . We then
 226 algorithmically construct the k paths between appropriately defined sets of sources and sinks in
 227 $M_{\leftarrow} \cup M_{\rightarrow}$. Next, we carry out an asymmetric shortcutting process, whose aim is to make sure that
 228 the new solution will be independent in the matroid, but also not disrupt the color structure. This
 229 allows us to prove that the paths following this postprocessing satisfy Definition 3.2. \square

230 **Theorem 3.4** *There is a randomized polynomial-time algorithm for FMMSM parametrized by*
 231 *$\varepsilon \in (0, 1)$ that outputs a set $S \in \mathcal{I} \cap \mathcal{U}$ (i.e., independent and upper-fair) such that*

- 232 • $\mathbb{E}[|S|] \geq (1 - \varepsilon)N$ with a high-probability tail bound:
 233 for $\delta > 0$, $\mathbb{P}[|S| < (1 - \delta)(1 - \varepsilon)N] \leq \exp(-\Omega_\delta(N))$
- 234 • $\mathbb{E}[f(S)] \geq 0.499 \cdot \varepsilon \cdot \text{OPT}_{\text{MatInt}}$
- 235 • for every $c \in [C]$ we have $\mathbb{E}[|S \cap V_c|] \geq (1 - \varepsilon)\ell_c$
- 236 • with a high-probability tail bound on the total fairness violation:
 237 for $\delta > 0$, $\mathbb{P}[\text{fav}(S) > (1 + \delta)\varepsilon \sum_c \ell_c] \leq \exp(-\Omega_\delta(\sum_c \ell_c))$

238 where N is the maximum size of a set in $\mathcal{I} \cap \mathcal{U}$, and $\text{OPT}_{\text{MatInt}}$ is the maximum f -value of a set in
 239 $\mathcal{I} \cap \mathcal{U}$ (clearly we have $\text{OPT}_{\text{MatInt}} \geq \text{OPT}$ as $C \subseteq \mathcal{U}$).

240 We stress that S is upper-fair with probability 1, not only in expectation. We also remark that one can
 241 show a similar tail bound for every individual ℓ_c , though the right-hand side $\exp(-\Omega_\delta(\ell_c))$ may not
 242 be meaningful unless ℓ_c is large. On the other hand, no such bound can be shown for the f -value,
 243 which in the worst case can be concentrated on a single element of the universe.

244 We give a brief sketch; the full proof can be found in the full version (see the supplementary material).

245 *Proof sketch of Theorem 3.4.* As the first step, we compute a maximum-cardinality fair and indepen-
 246 dent set P , which may be done in polynomial time by Lemma 2.4. By invoking Lemma 3.3
 247 we can show that $|P| = N$. As the second step, we compute a high-value independent and
 248 upper-fair set Y_0 . Using the algorithm of Theorem 2.6 ([LSV10]) (with $\delta = 10^{-3}$) we get that
 249 $f(Y_0) \geq 0.499 \cdot \text{OPT}_{\text{MatInt}}$. We denote

$$k(Y) = \sum_{c \in [C]} \max(0, |P \cap V_c| - |Y \cap V_c|)$$

250 for any solution Y , and $k := k(Y_0)$ to shorten notation. We will perform a number I of iterations
 251 which will be $(1 - \varepsilon)k$ in expectation. More precisely, let us set $I = \lceil (1 - \varepsilon)k \rceil$ with probability
 252 $(1 - \varepsilon)k - \lfloor (1 - \varepsilon)k \rfloor$, and $\lfloor (1 - \varepsilon)k \rfloor$ otherwise.

253 We perform I iterations. In the i -th iteration, we apply Lemma 3.3 to Y_{i-1} (and P) to obtain a
 254 collection $X_i^1, \dots, X_i^{k(Y_{i-1})}$ of augmenting or alternating sets. We choose one of them, X_i , uniformly
 255 at random, and apply it to obtain a new solution $Y_i = Y_{i-1} \triangle X_i$. Finally, we return $S := Y_I$.

256 All solutions Y_0, \dots, Y_I are independent and upper-fair. The properties of fairness lower bounds
 257 intuitively follow because at every iteration one random fairness violation is removed, and the

number of iterations is $\approx (1 - \varepsilon)$ times the initial number of fairness violations $k = k(Y_0)$. Since at least $|P| - |Y_{i-1}|$ of the sets at iteration i are augmenting, the claim about the size of the solution follows similarly. We can also show tail bounds by invoking Chernoff-Hoeffding style bounds for hypergeometric distributions. As for the objective value, we prove that at every step, the expected loss is only a $1/(k - i + 1)$ fraction of the current f -value, as we select randomly from among $k - i + 1$ disjoint augmenting or alternating sets. After $I \approx (1 - \varepsilon)k$ iterations we then end up with a telescoping product that simplifies to $\frac{\varepsilon k}{k} f(Y_0)$. \square

3.1 Deterministic algorithm

Now we turn to our deterministic result, Theorem 1.4. It is powered by a lemma that is an analogue of Lemma 3.3. Their proofs are deferred to the full version (in the supplementary material).

Lemma 3.5 *For any two matroids $\mathcal{I}_1, \mathcal{I}_2$, let $Y, P \in \mathcal{I}_1 \cap \mathcal{I}_2$ be two sets in their intersection, with $|Y| \leq |P|$. Then we may find in polynomial time a collection $X_1, \dots, X_{|P|-|Y|}$ of disjoint subsets of $Y \cup P$ such that for each set X_i we have $Y \triangle X_i \in \mathcal{I}_1 \cap \mathcal{I}_2$ and $|Y \triangle X_i| = |Y| + 1$.*

4 Experimental evaluation

We evaluate the performance of our algorithms empirically against prior work and natural baselines closely following the experimental setup of prior work [EMNTT20; EFNTT23], on a suite of benchmarks that are standard in the field: graph coverage, clustering, and recommender systems, under different fairness and matroid constraint settings. Our metrics are the submodular objective value $f(S)$ and total fairness violation $\text{fav}(S)$. All of the considered algorithms return sets that are independent and upper-fair, so the measured fairness violations are all with respect to the lower bounds. We compare the following algorithms:

- **OUR(ε)** – our algorithm of Theorem 3.4, for a range of settings of $\varepsilon \in \{0.2, 0.5, 0.8\}$. To compute a high-value solution Y , we run the natural greedy algorithm, which obtains a $1/3$ -approximation (Theorem 2.5), as the local search algorithm of Theorem 2.6 is impractical. The large fair set P is obtained via augmenting paths, ignoring f .
- **TWOPASS** – the algorithm of [EFNTT23] (Theorem 1.1). Since it was originally developed for the streaming setting, to get a fair comparison we simplify away the parts (namely the first pass) whose purpose was ensuring low memory usage. The first step of the algorithm obtains a fair set via augmenting paths (ignoring f). This is then divided in two, and each half is extended to an independent and upper-fair solution using a matroid intersection subroutine. For this we employ the greedy algorithm (the original implementation of [EFNTT23] used a swapping algorithm to ensure low memory and linear runtime, but it obtains inferior values).
- **LBMI** (Lower Bound Matroid Intersection) – an algorithm that always returns a fair set, with no theoretical guarantee on the value. It starts by building a fair set via augmenting paths, ignoring f , and then extends to a maximal solution using the greedy algorithm.
- **UBMI** (Upper Bound Matroid Intersection) – an algorithm that ignores lower bound constraints and just solves the matroid intersection problem for \mathcal{I} and \mathcal{U} (similar in spirit to MATROID-INTERSECTION from [EFNTT23]). Also here we use the greedy algorithm.
- **RANDOM** – an algorithm that randomly shuffles the universe and then adds each element if this keeps the solution independent and upper-fair.

For a fair comparison of the main underlying ideas, we made sure that the compared algorithms, particularly OUR and TWOPASS, use the same subroutines for similar tasks; for example, the implementations could likely benefit from heuristically taking f into account rather than ignoring f when building large fair sets, or from some local-search based postprocessing of the final solution. We do not compare to the algorithm of [ETNV24] (Theorem 1.2) as solving the multilinear extension makes it impractical. We repeat the randomized algorithms 40 times. All experiments can be run on commodity hardware (CPU only, single-threaded; we do not report runtimes) and take several hours to finish. Our code is in the supplementary material.

We outline the experimental scenarios below. In each experiment we vary a solution size scaling factor r , which roughly corresponds to the rank of the matroid \mathcal{I} .

Graph coverage. We use the Pokec social network [LK14]. Given a digraph $G = (V, E)$ of users and their friendships, we select a subset $S \subseteq V$ to maximize coverage, defined by

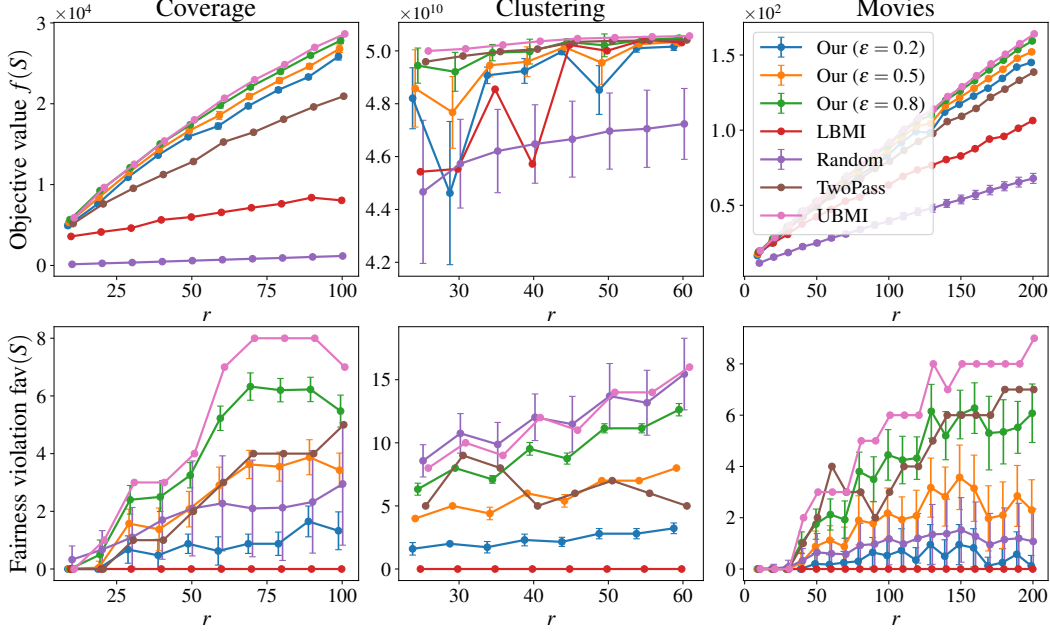


Figure 1: Our experimental results. Each column corresponds to one experiment; the top plot shows the objective value of each algorithm for a range of solution scale factors r , and the bottom plot shows fairness violations. For randomized algorithms we report averages, with error bars that correspond to sample standard deviation.

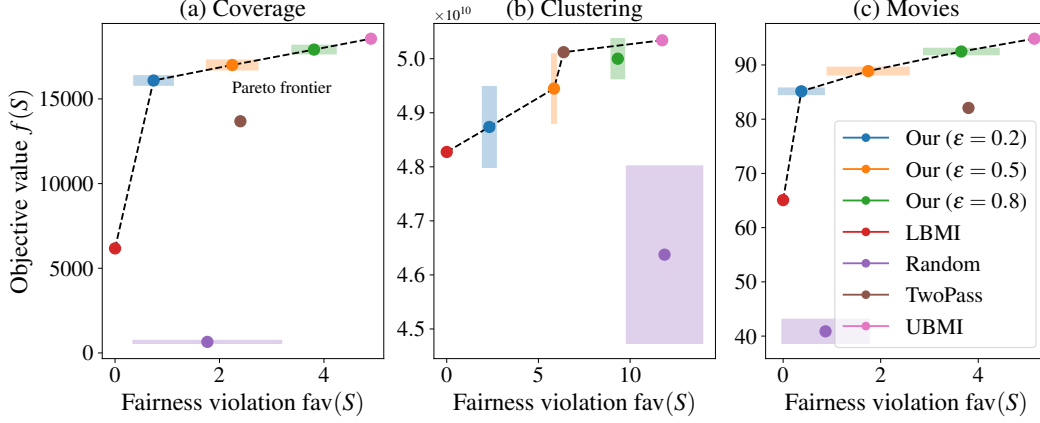


Figure 2: For each experiment and algorithm we take the average objective value and fairness violation over all r -values, and plot this as a single point. For randomized algorithms, the colored rectangles correspond to standard deviations. The dashed line corresponds to the Pareto frontier of the trade-off between objective value and fairness violation.

310 $f(S) = |\bigcup_{v \in S} N(v)|$, where $N(v)$ is the neighborhood of v . User profiles include age, gender,
311 height, and weight. We impose a partition matroid on body mass index (BMI). Profiles miss-
312 ing height or weight or with implausible data are removed, yielding a graph with 582,289 nodes
313 and 5,834,695 edges. Users are partitioned into four BMI categories (underweight, normal, over-
314 weight, obese), with upper bounds $\lceil \frac{|V_i|}{|V|} r \rceil$ for each group V_i . We also enforce fairness by age,
315 with 7 groups: $[1, 10]$, $[11, 17]$, $[18, 25]$, $[26, 35]$, $[36, 45]$, $[46, +]$, no age. We set $\ell_c = \lfloor 0.9 \frac{|V_c|}{|V|} r \rfloor$ and
316 $u_c = \lceil 1.5 \frac{|V_c|}{|V|} r \rceil$. We use r from 10 to 200.

317 **Exemplar-based clustering.** We use a dataset of 4521 phone calls from a Portuguese bank mar-
318 keting campaign [MCR14]. The goal is to select a representative subset $S \subseteq V$ for service quality
319 assessment. Each record $e \in V$ is represented as $x_e \in \mathbb{R}^7$ using 7 numeric features, includ-
320 ing age and account balance. We impose a partition matroid on account balance, with 5 groups:

321 $(-\infty, 0), [0, 2000), [2000, 4000), [4000, 6000), [6000, \infty)$. Each group V_i has upper bound $r/5$. Fair-
 322 ness is enforced by age, with 6 groups: $[0, 29], [30, 39], [40, 49], [50, 59], [60, 69], [70+]$, and bounds
 323 $\ell_c = 0.1r + 2, u_c = 0.4r$ for each c . We maximize the monotone submodular function [GK10]:
 324 $f(S) = \sum_{e' \in V} (d(e', 0) - \min_{e \in S \cup \{0\}} d(e', e))$ where $d(e', e) = \|x_{e'} - x_e\|_2^2$ and x_0 is the origin.
 325 We use r from 30 to 60.

326 **Recommender system.** We simulate a movie recommendation system using the MovieLens 1M
 327 dataset [HK16], with about one million ratings for 3900 movies by 6040 users. As in prior work [MB-
 328 NTC17; NTMZMS18; EMNTT20; EFNTT23], we compute a low-rank completion of the user-movie
 329 matrix [TCSBHTBA01], yielding $w_u \in \mathbb{R}^{20}$ for each user u and $v_m \in \mathbb{R}^{20}$ for each movie m . The
 330 product $w_u^\top v_m$ estimates user u 's rating for movie m . For user u , the monotone submodular utility
 331 for a set S of movies is $f(S) = \alpha \cdot \sum_{m' \in M} \max(\max_{m \in S} (v_m^\top v_{m'}), 0) + (1 - \alpha) \cdot \sum_{m \in S} w_u^\top v_m$,
 332 with parameter $\alpha = 0.85$ balancing coverage and personalized user score. We enforce proportional
 333 representation of movies by release date using a partition matroid with 9 decade groups (1911–2000),
 334 with upper bounds $\lceil 1.2 \frac{|V_d|}{|V|} r \rceil$ for each decade V_d . Movies are also partitioned into 18 genres c
 335 (colors), with fairness bounds $\ell_c = \lfloor 0.8 \frac{|V_c|}{|V|} r \rfloor$ and $u_c = \lceil 1.4 \frac{|V_c|}{|V|} r \rceil$. We use r from 10 to 200.

336 4.1 Results and discussion

337 Our results are depicted in Figs. 1 and 2. Similarly as prior work, we observe that enforcing fairness
 338 does come at some cost in the utility value, and that the utility values of the algorithms are much
 339 better in practice than the theoretical bounds guarantee.

340 In all three experiments, our algorithms produce solutions whose value is relatively competitive
 341 with UBMI, which completely ignores the lower bound constraints and accordingly has the highest
 342 fairness violations. In two of the three scenarios (coverage and movies), all OUR algorithms produce
 343 a higher f -value than all the other baselines (RANDOM, LBMI, and TWOPASS); in particular,
 344 TWOPASS is dominated by both OUR(0.2) and OUR(0.5) with respect to both metrics. For clustering
 345 the situation is somewhat unclear, but TWOPASS generally does better. In terms of violation of the
 346 lower bound fairness constraints, our different settings of ε , as expected, provide a smooth tradeoff.
 347 The baseline that guarantees no fairness violations, LBMI, does relatively poorly in terms of f -value.

348 This tunability of ε is a key strength of our approach, allowing users to select an operating point that
 349 best matches their specific requirements for the balance between utility and fairness.

350 5 Conclusion, limitations, broader impact, and future work

351 In this work we gave an improved algorithm for FMMSM which, for any $\varepsilon > 0$, returns an approxi-
 352 mate solution that satisfies an expected $(1 - \varepsilon)$ fraction of each fairness lower bound while satisfying
 353 the matroid constraint and the fairness upper bound constraints exactly; the returned solution is also
 354 large in size and enjoys high-concentration guarantees. Recent studies have shown that automated
 355 algorithms used in decision-making can introduce bias and discrimination. We make progress towards
 356 mitigating such effects in problems that can be formulated as submodular maximization under a
 357 matroid constraint, which are relevant to a range of applications such as forming representative
 358 committees or curating content for news feeds. We show the strong performance of our algorithm
 359 empirically on several real-world tasks. As in prior work, we observe a balance between fairness
 360 and utility value; however, this “price of fairness” should not be interpreted as fairness leading to
 361 inferior outcomes, but rather as a trade-off between two valuable metrics. The parametric nature of
 362 our algorithm (the tunable ε parameter) provides a new tool to help in navigating this balance.

363 Our work leaves open the exciting question of the approximability of FMMSM (without violations of
 364 fairness constraints) and MSPM. Is there a constant-factor approximation algorithm for MSPM? Or is
 365 there a superconstant hardness of approximation for FMMSM? (As remarked in [ETNV24], the latter
 366 result would give a negative answer to a fundamental question posed by Vondrák [Von13].) We also
 367 do not consider non-monotone objective functions or the streaming setting in this work. Finally, it is
 368 important to note that the fairness notion we employ, though standard and general, does not capture
 369 some notions of fairness considered in the literature (see e.g. [CR18; TWRTZ19]). No universal
 370 definition of fairness exists; the choice of which definition to apply is application-dependent and an
 371 active area of research.

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490 **A Supplementary Material**

491 We provide a full version of the paper with all omitted content, skipped proofs etc. together with the
492 supplementary material.

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